Determining Salinity Recharge Time for Deep Borehole Disposal - 17480

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ABSTRACT

It is generally accepted that the borehole above the disposal zone of a deep borehole disposal must be sealed 'completely' and 'permanently'. By completely it is meant that the seal zone overall must not allow fluids to flow up through it more readily than they could through the surrounding geological barrier. By permanently it is meant that the seal(s) must maintain their function(s) for long enough to satisfy the safety case which could be in excess of 100,000 years.

However, designing seals to meet these exacting requirements may be a case of over-engineering because the perturbations in the salinity gradients and density stratification of the groundwater created in and around the borehole during the drilling and operational phases of the borehole would be transient. Darcy flow of groundwater and diffusional flows will re-establish the salinity gradient which is one of the main barriers to the upward flow of fluid through the borehole and surrounding rock, restoring the natural barrier and making the engineered seals redundant.

In this paper we address the question of the timescale over which this recharge can be expected to take place. Our methodology involves establishing a mathematical model whose solution gives the time taken for an initial pressure disturbance to reach a steady state and the migration rates of brines resulting from this pressure field. We discuss the model, its assumptions and the solutions, both analytical and numerical.

INTRODUCTION

Deep Borehole Disposal (DBD) is an alternative disposal route to mined repositories for managing the high-activity, moderate-volume components of radioactive waste inventories [1-2]. It entails emplacement of waste packages in the lower portions of large diameter (up to 0.66 m) vertical holes drilled 4 to 5 km into the crystalline basement of the continental crust. This is around an order of magnitude deeper than the depths typically envisaged for a mined repository. DBD is also a multi-barrier concept, in which engineered barriers, including the wasteform, container and possibly backfilling materials, contribute to operational and long-term safety. However, the main safety emphasis is on the more robust geological barrier compared to mined repositories such as SKB's KBS-3 concept (on which the UK's proposed Geological Disposal Facility (GDF) is also based).

At the depths being considered for DBD, lateral movement of groundwater is normally limited due to very low bulk hydraulic conductivities while upwards movement of potentially contaminated groundwaters is further constrained by a density stratification (resulting from a natural salinity gradient) that has led to isolation from near-surface waters. This isolation can have prevailed for millions of years. However, to take advantage of the isolation provided by this natural geological and hydrological barrier the waste packages and the disposal zone have to be sealed off so that the borehole itself does not provide an easier route back to the biosphere for fluids (potentially contaminated by any radioactive materials which might leach out following corrosion of the waste containers) than does the surrounding geology. At the very least, this requires that the borehole above the disposal zone is completely closed off to upwards flow by impermeable seals and, ideally, within the disposal zone itself the annulus between the borehole casing and borehole wall and that between the casing and the waste packages should also be sealed to minimise fluid movements.

Several sealing methods and materials have been proposed for closing off the borehole above the disposal zone, including: cement, asphalt and swelling clays [3]. However, experience from attempts to seal wells in the oil and gas and geothermal industries suggests than none of these methods is likely to be successful on the timescale required by the safety case for radioactive waste isolation. Furthermore, none of these sealing methods will avoid the disturbed rock zone around the borehole (created during the drilling) potentially short circuiting the seals and allowing contaminated groundwaters to reach the human environment. One method for sealing the borehole that has the potential to prevent this by eliminating the disturbed rock zone is rock welding [4]. This is a technology, under development at the University of Sheffield, in which a short section of the casing is removed from the borehole above the disposal zone. A sacrificial electric heater, powered from the surface, is placed in this section surrounded by crushed granitic host rock and used to partially melt the crushed rock and the host rock for an appropriate distance beyond the borehole wall. When sufficient melting has occurred power is reduced and the melt allowed to cool and recrystallize to solid granite continuous with the host rock. These ‘welds’ can be repeated at intervals with the gaps between backfilled with other sealing materials.

Irrespective of how the borehole is sealed, for the post-closure performance assessment it is normally considered necessary to demonstrate that the integrity of the seals will survive on the time scale deemed appropriate for containment of the wastes. For many high-activity wastes such as spent fuel and HLW this could be for 10^5 to 10^6 years.

During the drilling of the borehole, usually with fresh water, brine or an aqueous mud, the natural salinity gradient and resulting hydrostatic pressure gradient prevailing in the host rock will be disrupted. This will be greatest in the hole itself but will also affect the wall rock adjacent to the borehole and ‘tail off’ out into the more distant regions of the host rock. The nature and extent of the disruption will depend on circumstances but will continue throughout the operational phase until the disposal zone is filled, the borehole is sealed and human activity ceases. However. Once this point is reached, natural forces will begin to try to restore the regional salinity and pressure gradients in and around the borehole and eliminate the perturbation. Once this is achieved the natural barrier to upwards flow of fluids will be re-established in and around the borehole just as it is in the surrounding rock. This would effectively render the engineered seals in the borehole superfluous, especially after any thermal 'high' superimposed on the geothermal gradient as a consequence of radioactive decay of the wastes has passed.

It is therefore a matter of considerable importance in defining the period for which the main engineered seals in a deep borehole disposal have to fulfill their function to be able to ascertain how long it would take to re-establish the regional salinity and pressure gradients in the borehole. In this paper we attempt to address this issue through physical and numerical modelling.

METHODOLOGY

Physical Model

We consider a borehole drilled to a depth of 5 km in the crystalline basement which is then subsequently filled with fresh water. The creation of the borehole generates a pulse in pressure arising from the (depth dependent) difference in pressure between fresh water and the brines present in the surrounding rock. The pressure pulse will travel by a diffusive mechanism and eventually a steady state distribution will become established. Any pressure differences will give rise to movement of brine towards the borehole. Darcy flow will continue until the density profile inside the hole generates a pressure equal to that on the outside. Thus there are two physical processes occurring, each with a different timescale; Darcy flow of brines through the fractured rock driven by time dependent pressure differences, and re-establishment of the salinity gradient within the borehole – driven by gradients in chemical potential, temperature and pressure. Once the first containers of nuclear waste are emplaced in the borehole, an additional complication arises from the need to consider the effect on the local temperature due to sources of heat.

A logical first step in solving this complex problem is to decouple the pressure changes resulting from the initial pulse from the mass transport within the hole, and to treat the borehole as a column of water without any waste packages, casing, seals, and support matrices.

Fluid pressure

The pressure inside of the borehole depends on the local density of water through the relation

$$p = r g z \tag{Eq. 1}$$

Where ρ is the density, z is depth and g is the gravitational acceleration. To a good approximation, the pressure increases by 9.792 kPa for every meter increase in depth. Temperature also increases with depth from the geothermal gradient. To determine the local borehole pressure we have used thermodynamic data tables to interpolate the density based upon the geothermal temperature and pressure at a given depth. The local pressure is then obtained using Eq. 1 with the average density replacing the absolute density (the average density is obtained by taking the arithmetic mean of the densities evaluated at the lower depths). For the intrarock brine, the local density is adjusted to take into account the dissolved solids present at each depth, after which the pressure is once again computed using Eq. 1 with an average density. In this work we have used the temperature and mass of dissolved solids at various depths based on the Urach 3 geothermal borehole in Germany [5].

The difference between rock and borehole fluid pressure is plotted as a function of depth in Fig. 1. The pressure difference is close to zero at depths up to 1000 m, afterwhich, it increases quadratically with depth. If we ignore the points above 1000 m, the variation in pressure with scaled depth ([z-1000]/L, where L is now 4000 m) can be fitted to the following 2nd order polynomial:

$$Dp = p_0\bar{z} + a\bar{z}(\bar{z} - 1); \quad \bar{z} = (z - 1000)/L \qquad \text{(Eq. 2)}$$

with best fit coefficients determined to be: p_0 = 2.319(3) MPa, a = 2.17(1) MPa, (numbers in parentheses are the uncertainties in the last digit).

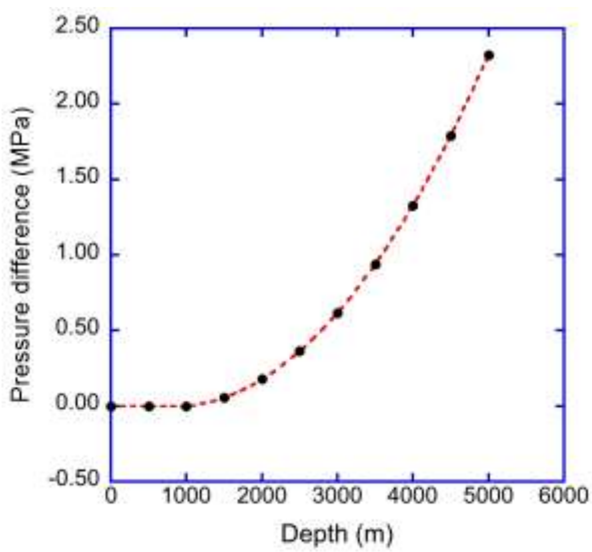

Fig. 1. Plot of the difference in pressure between a column of fresh water and a column of brine as a function of depth. The dashed line is drawn for guidance only.

Mathematical Model

The flow of fluid through a porous solid driven by a pressure gradient is described by Darcy's law:

$$\boldsymbol{q} = -\frac{k}{\mu}\nabla p \qquad \text{(Eq. 3)}$$

in which ***q*** is the flux (or equivalently, the volumetric flow rate per unit area or discharge velocity), k is the permeability of the porous medium and μ the dynamic viscosity of the fluid. When this equation is combined with the equation of continuity, an equation describing the motion of a pressure pulse results ('pressure diffusion' equation):

$$\nabla^2 p = \frac{1}{k}\frac{\partial p}{\partial t} \qquad \text{(Eq. 4)}$$

where κ is known as the hydraulic diffusivity. At infinite time, the right hand side of Eq. 4 vanishes and the solution of Laplace's equation with appropriate initial and boundary conditions then gives the steady state pressure distribution throughout the rock. In writing Eqs. 3-4 we have assumed that the rock is homogeneous and that a single scalar permeability can be used throughout. Similarly, we ignore the dependence of the fluid viscosity on salinity and thermodynamic state.

Because the hydrostatic pressure (Eq. 1) satisfies Laplace's equation we shall henceforth take p to be the pressure relative to the hydrostatic distribution.
In a coordinate system in which a borehole of length L and radius r_0 lies parallel to the z-axis with its terminal depth at $z = L$ and surface at $z = 0$, the boundary conditions are:

(i) $p = 0$ at $z = 0, L,$ for all time,
(ii) $p = p(z)$ at $r = r_0$, $0 \le z \le L$, for all time,

while the initial condition is:

(iii) $p = 0$ at $t = 0$ for $r > r_0$ and $0 \le z \le L$

The second part of condition (i) is unlikely to be true in a real borehole where some upward flow can be expected, however, taking p = 0 at z = L is mathematically convenient and probably justified for most values of the radial distance where the strong radial pressure differences would mean any axial flow is negligible in comparison to radial flow. Condition (ii) is certainly not true in practice since the instantaneous pressure distribution will change in time, including at the borehole wall. However, a time-independent boundary condition enables a tractable mathematical solution to be obtained which then serves as an upper bound on the more realistic time-dependent case.

RESULTS AND DISCUSSION

Steady State solution

After infinite time, the right hand side of Eq. 4 vanishes and we have Laplace's equation – the steady state condition. A solution of Laplace's equation in cylindrical polar coordinates can be obtained in terms of modified Bessel functions and Fourier series which satisfies the boundary conditions. The steady state pressure so obtained is:

$$p = \frac{2}{\pi} \sum_{n=1}^{\infty} C_n \frac{K_0\left(n\pi r / L\right)}{K_0\left(n\pi r_0 / L\right)} \sin\left(n\pi z / L\right) \qquad \text{(Eq. 5)}$$

where K_0 is a modified Bessel function of the second kind, order zero while C_n is given by

$$C_n = \begin{cases} -\dfrac{p_0}{n} & n = 2,4,.... \\ \dfrac{1}{n}\left(p_0 - \dfrac{4a}{\pi^2 n^2}\right) & n = 1,3,5.... \end{cases} \qquad \text{(Eq. 6)}$$

A set of steady state pressure contours is plotted in Fig. 2. More than 5000 m from the borehole wall, the pressure difference is essentially zero while the greatest pressure differences occur below 3000m in depth. These pressure differences will cause the flow of brines towards the borehole according to Darcy's law (Eq. 3).

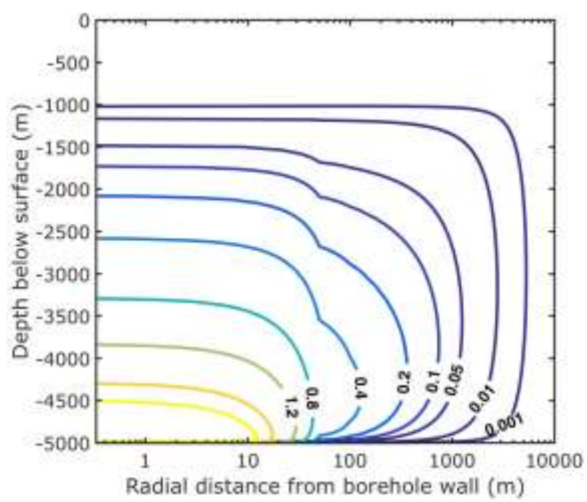

Fig. 2. Pressure difference contours (MPa) after infinite time (steady state).

The flow arising from this pressure field can be obtained by taking the spatial derivatives of Eq. 5 in the radial and axial directions, giving components of the discharge velocity: $\boldsymbol{q}$ = (u,v):

$$u = \frac{-2k}{\mu L}\sum_{n=1}^{\infty} nC_n \frac{K_1\left(n\pi r / L\right)}{K_0\left(n\pi r_0 / L\right)}\sin\left(n\pi z / L\right)$$
$$v = \frac{-2k}{\mu L}\sum_{n=1}^{\infty} nC_n \frac{K_0\left(n\pi r / L\right)}{K_0\left(n\pi r_0 / L\right)}\cos\left(n\pi z / L\right) \qquad \text{(Eq. 7)}$$

Where K_1 is a modified Bessel function of the 2nd kind, order 1. The radial and axial components of the discharge velocity are shown in Figs. 3-4 at four different depths. We have taken a value of permeability representing the high end of available measurements for fractured granites of 10^{-16} m^2 and a value for the viscosity of water of 0.2818 mPas (cP). We make no attempt to allow for variations in viscosity for salinity, temperature or pressure in the present work.
Fig. 3 shows that the fluid moves towards the borehole and the speed at which it moves increases with depth. The magnitude of the flow is 1-10 meter per year but this drops off quickly with radial distance. Fig. 4 shows that upward flow is negligible compared to the radial flow, being at least 2 orders of magnitude lower. The axial component changes sign at various depths but is negligible at around 10 m out from the borehole.

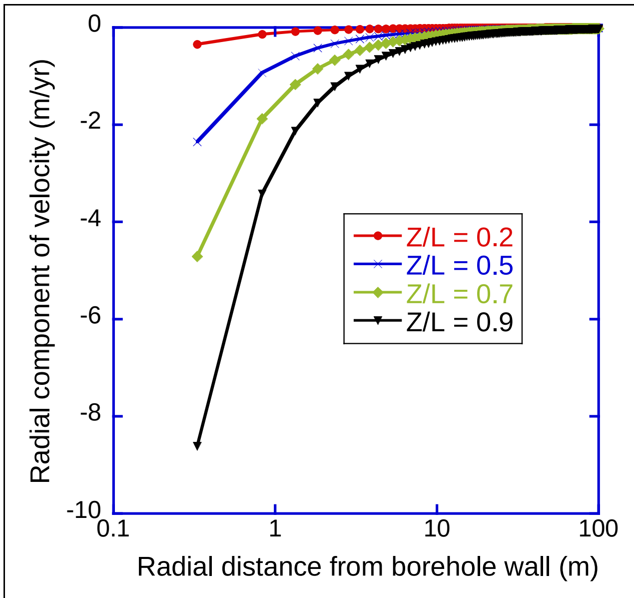

Fig. 3. Radial component of discharge velocity at scaled depths: Z/L = 0.2, 0.5, 0.7 and 0.9.

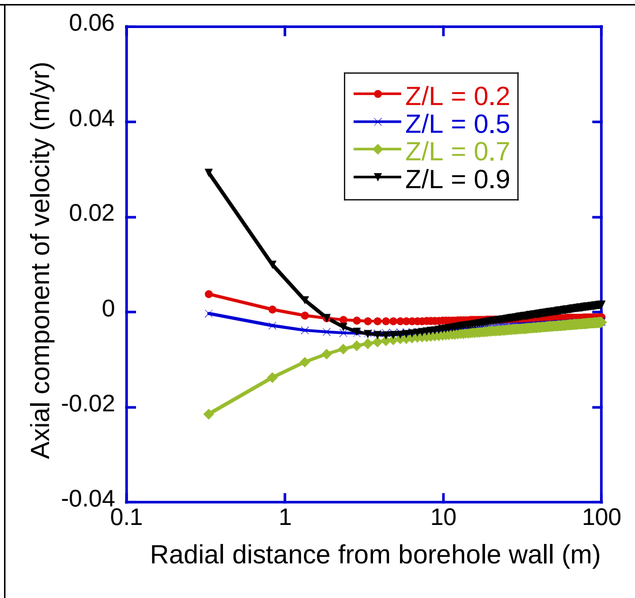

Fig. 4. Axial component of discharge velocity at scaled depths: Z/L = 0.2, 0.5, 0.7 and 0.9.

The time dependent problem

No analytical solution to Eq. 4 can be found which satisfies the boundary conditions for the full time dependent case. A numerical approach is therefore required.

As Eq. 4 is mathematically identical to the heat conduction equation, it can be satisfied by a 'point source' solution [6], which in the current case gives the pressure at a point ***R*** in space and at time *t* due to an instantaneous source of momentum placed at the origin.

$$p(\boldsymbol{R},t) = \frac{\psi}{8(\pi\kappa t)^{3/2}} \exp\left(-R^2 / 4\kappa t\right) \quad \text{(Eq. 7)}$$

where ψ is the strength of the point source and $R = |\boldsymbol{R}|$. By introducing point sources along the z-axis between $z_L < z < z_H$ and integrating with respect to z, one obtains an expression giving the pressure due to an instantaneous finite line source. A further integration with respect to time then gives an expression for the pressure due to a *continuous* finite line source. Taking a line source along the positive z-axis, together with an image line source placed at $-z_L < z < -z_H$, each with a quadratically varying strength per unit length, constant in time, an approximate model can be established which still satisfies a modified set of boundary conditions. The use of an image source guarantees that boundary condition (i) is satisfied ($p = 0$ at $z = 0$). The pressure is no longer zero at $z = L$, but is instead replaced by the condition that it vanishes at infinity. This change will have no adverse consequence at distances not too close to the z-axis and test comparisons we have conducted confirm this is indeed the case. We expect the solution of this model to give good agreement with the steady state analytical solution as long as the radial distance is not too small.

The pressure at time *t* at position (*r*,*z*) is given by

$$p(r,z,t) = \int_{z_L}^{z_H} \frac{j(l)}{\sqrt{r^2+(z-l)^2}}\, erfc\left(\sqrt{\frac{r^2+(z-l)^2}{4kt}}\right) dl \; + \int_{-z_H}^{-z_L} \frac{j(l)}{\sqrt{r^2+(z-l)^2}}\, erfc\left(\sqrt{\frac{r^2+(z-l)^2}{4kt}}\right) dl \quad \text{(Eq. 8)}$$

where φ is to be interpreted as $\varphi(-\lambda) = -\varphi(-\lambda)$ for negative values of λ. The hydraulic diffusivity, κ, is related to properties of the rock and fluid through

$$k = \frac{k}{f m c} \quad \text{(Eq. 9)}$$

where ϕ is the porosity of the rock, and c is the compressibility of the fluid, the product of the two being known as the storativity of the medium. Taking a storativity value of 10^{-9} together with the earlier mentioned values of permeability and dynamic viscosity gives a hydraulic diffusivity = 0.0004 m^2s^{-1}. The source strength per unit length, φ, in Eq. 8 is taken to be a quadratic function of depth:

$$j(l) = al + bl^2 \qquad \text{(Eq. 10)}$$

in which α and β are constants with dimensions of inverse distance. The value of these constants was obtained by noting that the pressure on the borehole boundary is given by Eq. 2 for all values of time. Letting $t \to \infty$ infinity in Eq. 8 and integrating the resulting expression gives the steady state pressure distribution for the finite line source (and its image). The borehole wall ($r = r_0$) corresponds to $r = 0$ for the line source. However, the solution is not valid for $r = 0$ and so we simply take an arbitrarily small value of r (0.001 m) and equate this pressure to that of Eq. 2, allowing values of α and β to be determined by a least squares fit procedure. We find these values are: α = -6.131 Pa m^{-1} and β = 0.004 Pa m^{-2}.

The pressure defined by Eq. 8 is plotted as a function of time in Figs. 5-8, each figure corresponding to one of four values of $\bar{z}$ = 0.25, 0.5, 0.75 and 1.0, for four different values the radial distance from the line source: r = 1 m, 10 m , 100 m and 1000 m.

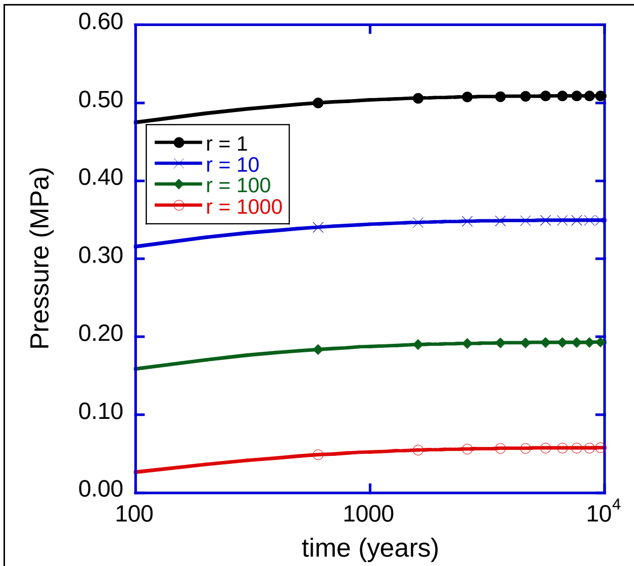

Fig. 5. Pressure versus time for the finite line source model at $\bar{z}$ = 1, r = 1m, 10m, 100m and 1000m.

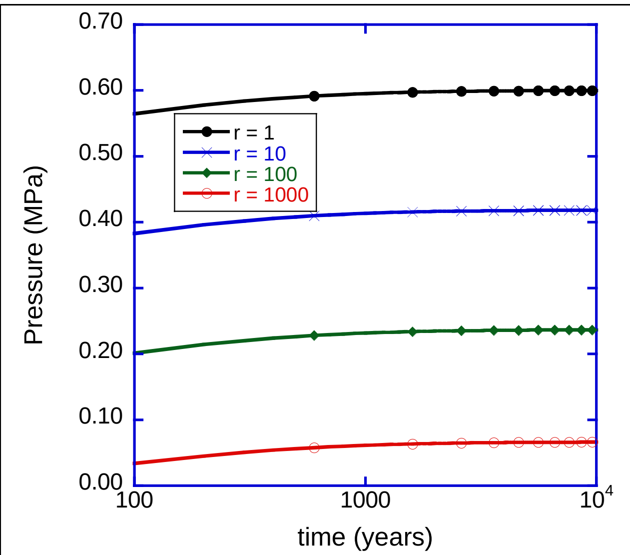

Fig. 6. Pressure versus time for the finite line source model at $\bar{z}$ = 0.75, r = 1m, 10m, 100m and 1000m.

The results from Figs. 5-8 show that the pressure always reaches a plateau value no matter which values of r and z are considered. Pressure diminishes with increasing radial distance from the line source regardless of z, but there is a slight anomaly in that the pressures are slightly greater close to the line source at 4000 m depth (Fig. 6) than they are at 5000 m depth (Fig. 5), which might indicate that a more careful treatment of the bottom boundary condition might be needed for the line source model. Once a depth of 2000 m is reached (Fig. 8) the pressure is lower by an order of magnitude reflecting the greatly reduced density differences between the fresh water in the borehole and that in the surrounding rocks at this depth. The time taken to reach the steady state pressure at this lower depth is also reduced by an order of magnitude compared to the other cases. For $\bar{z}$ = 0.25, it takes approximately 1000 years for the pressure to reach its steady state value compared with around 10,000 years at the lower depths.

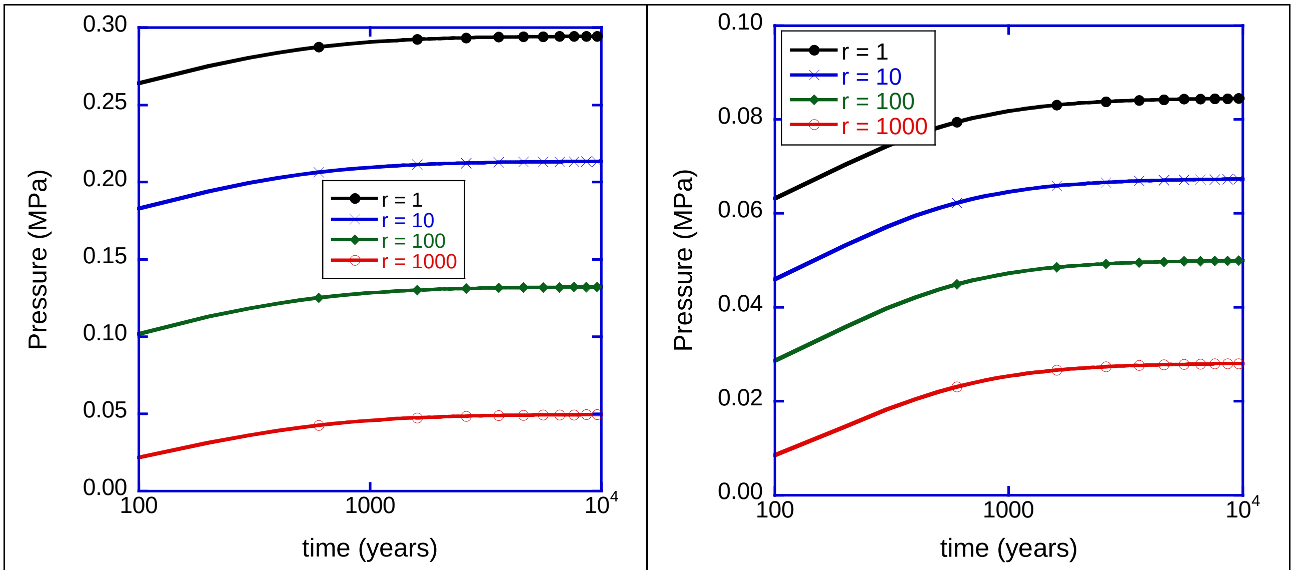

Fig. 7. Pressure versus time for the finite line source model at $\bar{z}$ = 0.5, r = 1m, 10m, 100m and 1000m.	Fig. 8. Pressure versus time for the finite line source model at $\bar{z}$ = 0.25, r = 1m, 10m, 100m and 1000m.

CONCLUSIONS

We have a developed a simple mathematical model which is able to predict the timescales over which an initial pressure disturbance, caused by differences in the pressure between the borehole fluid and that of the surrounding intrarock fluid. Our model involves the use of a pair of line sources (source and image) which enables a numerical solution to be obtained for the time dependent pressure diffusion equation.

Prediction of the timescale over which the pressure differences approach steady state is the first step towards estimating the time taken to re-establish the salinity gradients within the borehole. Our model could easily be adapted to solve the more complicated diffusion part of the full problem.
Results obtained using this model for a 5 km deep borehole and a realistic salinity gradient suggest that it will take on the order of 10,000 years for the flow pressure to approach a steady state; the time to re-establish the salinity gradient will be somewhat longer than this due to the diffusion processes which would then take place within the borehole.

All mathematical models contain a certain number of assumptions and approximations and the present one is no exception. The constant (in time) pressure condition on the borehole axis is perhaps the greatest approximation. This condition could be relaxed through use of an iterative approach in which the boundary condition is continually updated using the most recent solution of the pressure diffusion equation. Use of a state dependent viscosity for the brine could be added as well as a more realistic model for the fractured granite (permeability may change with location). We have used representative values for the various material properties. With the establishment of the Deep Borehole Field Test [7] in the USA, it should be possible to obtain more realistic values of these parameters in the near future, enabling a more robust validation for the model.

The present work, while only the first step of a larger modelling program indicates that none of the current proposed sealing methods (with the exception of rock welding, which is still at an stage of development) are likely to last for the length of time required by the post closure safety case.

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